## NOTATION

$A$ = constant of integration
$Ea$ = apparent energy of activation, B.t.u./(lb. mole) (°R.)
$F$ = feed rate, moles methane/hr.
$T$ = temperature, °R.
T.F. = time factor, lb. catalyst/mole methane fed/hr.
$W$ = weight of catalyst, lb.
$(CH_4)$ = mole methane
$k$ = reaction velocity constant, moles/(hr.) (lb. of catalyst) (atm.)
$pCH_4$ = partial pressure of methane, atm.
$r$ = reaction rate, moles/(hr.) (lb. of catalyst)
$x$ = conversion, moles/mole

### Subscript

$F$ = feed

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# Graphical Solutions on Dimensionless-number Plots


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**A graphical method for solving problems involving dimensionless-number plots is presented, and its application demonstrated for the relatively involved case of liquid cyclone scale-up.**


Dimensionless-number plots frequently are awkward to use, in that the dependent variable, or unknown, may be contained in more than one of the dimensionless-number groups. In such a case there has been no direct way to solve for the unknown. Before the unknown could be evaluated, roundabout calculations have had to be made in order to locate the appropriate point on the dimensionless-number plot.

By way of example the familiar log-log plot of drag coefficient vs. Reynolds number, shown as Figure 1, may be considered. This relates to the free settling of spherical particles in a fluid. In actual problems which arise the unknown is most frequently either terminal settling velocity $u_p$ or particle size $D_p$, both of which are included in each of the two dimensionless groups. When either $u_p$ or $D_p$ is unknown, the value for $Re$ cannot be calculated directly, and neither can the value for $C_R$. Neither coordinate on the dimensionless-number graph, therefore, can be evaluated directly, and further means must be employed to find where on the graph the problem operation should be plotted.

One method for solving sedimentation problems when either $u_p$ or $D_p$ is unknown involves replotting the information of Figure 1 on a graph of $Re$ vs. a new dimensionless function $C_R Re^n$, the exponent $n$ being so chosen that the unknown variable, either $u_p$ or $D_p$, cancels out of the function. Since the new function does not contain the unknown, it can be evaluated directly. The corresponding value of $Re$ can thus be determined by means of the new graph, and the unknown calculated from the discovered value of $Re$.

There is also a graphical construction (*1*) for discovering the appropriate value of $Re$, in which the function

$$\log C_R = -n \log Re + \log C_R Re^n \quad (1)$$

is plotted in Figure 1. Here again the exponent $n$ is chosen so that the unknown cancels from the final term, which can therefore be evaluated. The intersection of the plot of Equation (1) with that originally constituting Figure 1 locates the required $Re$ value, namely, $Re'$, from which the unknown can be calculated.

Neither of the methods described provides a graphical element to measure the value of the unknown. In this paper the graphical method will be elaborated to permit representing the unknown directly, and the particular utility of the method will be demonstrated in connection with solid-liquid cyclone scale-up problems.

## GRAPHING AS A VECTOR ADDITION

Graphing, at least in Cartesian coordinates, may be regarded as a vector addition. Each variable on a graph has been assigned not only a scale, but also a direction. Thus the value of each variable is represented in the graph by a vector. To plot a point (*see Figure* 2), one combines these chart vectors in typical vector fashion, starting at the origin. The outer end of the resultant is marked by the plotted point. In this view the axes are essentially only markers showing the direction and scale of the corresponding chart vectors.

The major variables of Figure 1 are $C_R$ and $Re$. Each comprises several original or subordinate variables, three of which, namely, $u_p$, $D_p$ and $\rho_f$, are included in each of the major variables. It can be shown that a chart direction and scale exist for each of the subordinate variables and further that a point which is plotted by summing the chart vectors for the two major variables (the normal plotting of $C_R$ vs. $Re$) may also be plotted by summing chart vectors correponding to the subordinate terms comprising them. By way of example it will be demonstrated that there exists a direction and scale corresponding to the variable $u_p$.

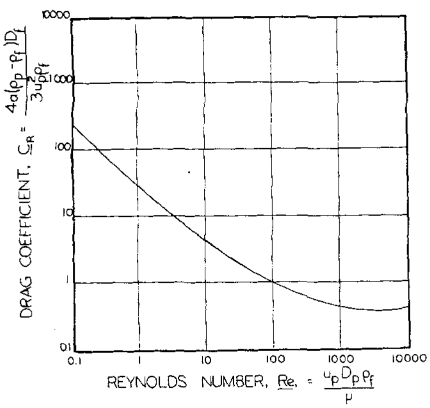


Fig. 1. Drag coefficient vs. Reynolds number for settling particles.

When $u_p$ is eliminated between equations defining $C_R$ and $Re$, it follows that

$$\log C_R = -2 \log Re + \log \frac{4a\,(\rho_p - \rho_f)\,D_p^3\,\rho_f}{3\mu^2} \qquad (2)$$

[This is Equation (1) with a value of 2 for the exponent $n$.] As the scales of Figure 1 are logarithmic, it will be seen that (2) is the equation of a family of straight lines with a slope of —2, the position of which depends upon the value of the parameter $4a(\rho_p - \rho_f) D_p^3\rho_f/(3\mu^2)$. As $u_p$ is varied with all other subordinate variables held constant, the plotted point must always move along a straight line with a slope of —2. Therefore there is a chart direction corresponding to changes in value of the variable $u_p$.

There must be a scale corresponding to $u_p$ proportional in module to that of $Re$, so that its projection upon the $Re$ axis has the same module as the $Re$ scale. This follows from the definition $Re = u_p D_p \rho_f/\mu$. If $u_p$ is increased by a given factor (log $u_p$ increased by a given amount and other variables held unchanged), then $Re$ must change by the same factor (log $Re$ increases by the same amount).

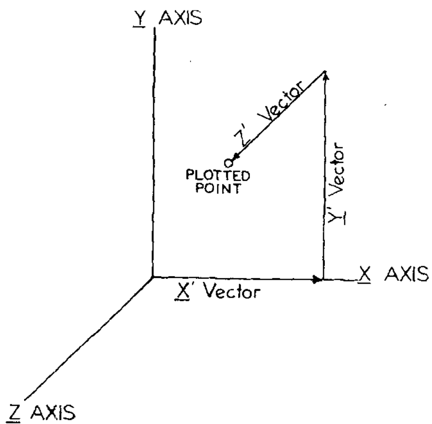


Fig. 2. Plotting as a vector operation.

As a chart direction and scale may be assigned to $u_p$, it may be represented by a chart vector and an axis drawn indicating this direction and scale.

In an analogous manner it can be shown that there exists a $D_p$ axis with a slope of +1, and a $\rho_f$ axis with a slope of —1, both having scales which when projected upon the $Re$ axis are commensurable with the $Re$ scale. [Note that $\rho_f$ and $(\rho_p - \rho_f)$ are treated as though independent of one another; that is, the $\rho_f$ chart vector will not take into account any change produced in the variable term $(\rho_p - \rho_f)$. This is treated as a separate entity.] The $(\rho_p - \rho_f)$ and $a$ axes coincide in direction and scale with the $C_R$ axis, as these variables do not appear in $Re$. The constant term 4/3 conventionally introduced into $C_R$ must also be measured in this direction. The $\mu$ axis has the same scale module as that of $Re$ but is rotated 180° from the $Re$ axis.

Subordinate axes have been added on the conventional $C_R$ vs. $Re$ plot in Figure 3. Chart vector summations are shown for a set of values of subordinate variables (variables having values less than 1 and hence a negative log value are shown with dotted lines) and also for the corresponding values of the major variables. The two summations have the same resultant point. It will probably be obvious that they must but, if desired, this can be verified by summing projections of the subordinate vectors along each of the major axes in turn. Summing the projections of the subordinate variable vectors along the $Re$ axis and remembering that the thus-projected log scales of each of the subordinate vectors have the same module as the log $Re$ scale except that the log $\mu$ scale is reversed makes it obvious that

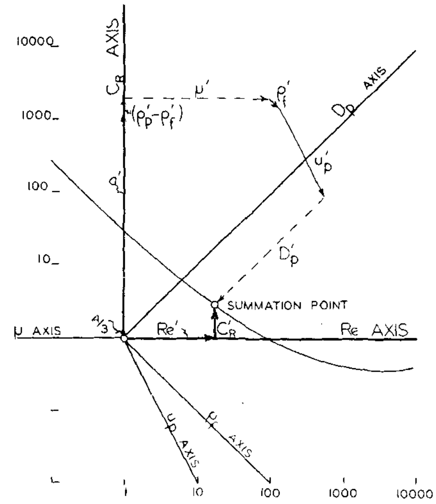


Fig. 3. Summation of major and subordinate vectors.

$$-\log \mu' + \log \rho_f' + \log u_p' + \log D_p'$$

should be equal to log $Re'$, which it is by identity.

The same method, taking into account the slopes of the various subordinate vectors, will show that the $C_R$ coordinate of the subordinate vector summation must equal the sum of the projections of the subordinate vectors along the $C_R$ axis.

In order to plot an unambiguous relationship between major variables, it is necessary that any plotted point be uniquely resolved into the chart vectors which were combined to locate the point. Therefore the directions or axes assigned to the different major variables must each have at least one directional component which is not shared by any of the others. This limits the number of major variables which can be fully plotted to the number of dimensions available for the graph. Points cannot be uniquely resolved into subordinate chart vectors, however, since these share graph dimensions.

Since the chart vectors for major variables always start at the coordinate zero on the corresponding scale, the coordinate of the terminal point measures the magnitude of the variable. Chart vectors for subordinate variables need not start at zero on their respective scales, and so the coordinate of the terminal point does not determine the length of the chart vector and therefore does not in itself determine the magnitude of the variable.

## GRAPHICAL SOLUTION BY COMPLETING VECTOR ADDITION

The idea of subordinate vectors can be used to determine directly and graphically the value of any one of the subordinate or original variables if the values of all the others are specified.

The summation of chart vectors must always terminate on an appropriate $C_R$ vs. $Re$ curve. Also the chart vectors may be combined in any order to give the same summation point. Therefore, if all but some one of the subordinate chart vectors are combined, the point located by this partial or incomplete summation will be off the curve by an amount determined by the magnitude and direction of the missing chart vector. The value of the missing variable is measured directly by the distance from the incomplete summation point to the curve in the direction assigned to the missing variable. This permits solving graphically for any one of the variables if the others are known.

*Example.* If the unknown or dependent variable is particle size $D_p$, the coordinates of the summation point for all variables except $D_p$ would be determined. The component of the summation incomplete by $D_p$ in the $C_R$ direction is equal to log $4a(\varrho_p-\varrho_f)/(3u_p^2\varrho_f)$, which is log $C_R'/D_p$. The component in the $Re$ direction is log $u_p\varrho_f/\mu$, which is log $Re'/D_p$. The closing vector corresponding to $D_p$ starts at this incomplete summation point, extends along the $D_p$ vector direction with a slope of +1, and terminates on the curve at point $C_R'$, $Re'$, the values of which can be read directly from the log scales provided on the respective axes. The value of log $D_p$ is then measured by the length of the closing vector, but since the projection of the $D_p$ scale on the $Re$ axis is commensurate with the $Re$ scale, the scale length of the $D_p$ vector can actually be determined from the $Re$ coordinates; that is, the value of $Re'$ is read from the log scale on the $Re$ axis, and the value of $Re'/D_p$ has already been determined. Obviously $D_p = Re'/(Re'/D_p)$.

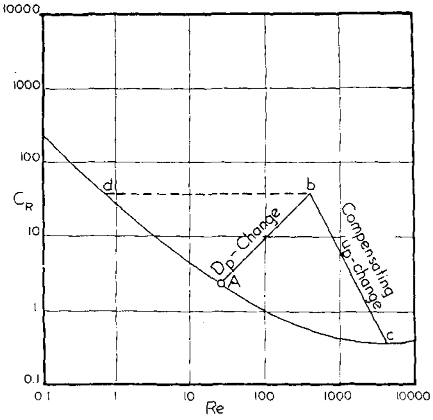

Fig. 4. Solution by compensating "change vectors."

## SOLUTIONS BY COMPENSATING "CHANGE VECTORS"

A type of problem to which the proposed graphical solutions are particularly applicable arises in scale-up operations. In such cases the behavior of a model system is known. When the value of one or more variables in the operation are changed, there must be a compensating change in one or more of the others. It is desired to evaluate this compensating change.

On the graphical constructions, when one variable is changed by a fixed factor, its vector is changed by a corresponding fixed length, since log (variable × factor) = log (variable) + log (factor). The change factor itself can be considered as contributing a vector which is to be added to the vector representing the original value of the variable. Such "change vectors" may be manipulated in the constructions as independent entities.

A change in the value of any one variable in an operation, by a given factor, adds a corresponding "change vector" to the summation point lying on the curve of the dimensionless-number plot. Compensating "change vectors" must then also be added corresponding to compensating changes in one or more of the other variables, in order to bring the summation point back onto the curve. The magnitude of these compensating changes can be evaluated from the lengths of the corresponding change vectors.

*Example.* Point $A$ in Figure 4 represents a certain known sedimentation operation. The size of the particles is to be increased by a factor of $z$. This adds a $D_p$-change vector Ab in the $D_p$ direction of such length that $Re_b$ is $z$ times $Re_A$. If this is to be compensated by a change in settling rate $u_p$, as it would be if the properties of the fluid and particles remained the same, then the compensating $u_p$-change vector to return to the curve would be $bc$ drawn parallel to the $u_p$ axis. Because of the previously discussed projection relationships, the length of the $u_p$ change vector corresponds to a change in $u_p$ by a factor of $Re_c/Re_b$.

Alternatively, if the change in particle size were to be compensated instead by a change in viscosity of the liquid $\mu$, then the compensating $\mu$-change vector would be $bd$, and the viscosity would have to be changed (because of the reversed scale of $\mu$) by a factor $Re_b/Re_d$.

## SCALE-UP BY COMPENSATING VECTORS

The use of compensating vectors for scale-up will be illustrated by showing how the method might be used to predict the change in particle-size separation produced in liquid-solid or hydraulic cyclones when the size or scale of the cyclone is changed. In setting up the calculations, many assumptions have to be made concerning the flow patterns in cyclones. There will be no attempt in this paper to justify any of the cyclone assumptions, since the object is to present the graphical methods and the purpose of bringing in cyclone problems is purely to demonstrate the graphical methods.

In a cyclone new feed is injected tangentially at the periphery of the unit and is displaced inward toward the axis in a spiral path. The solid particles are subjected to a centrifugal force generated by the tangential components of flow and an opposing centripetal drag force resulting from the inward radial components of flow. Solids which develop a sufficiently high setting rate under the influence of the centrifugal field are flung against the bounding walls of the cyclone, from whence they are swept toward the apex of the conical end in a current induced by the drag of the walls. There they are discharged as a concentrated suspension through an apex opening. Solids developing insufficient settling rates to overcome the radial components of flow toward the axis are dragged by the fluid in toward the axis of the cyclone. They exit as a partially clarified overflow through a cylindrical duct or "vortex finder" concentric with the axis in the flat, or nonconical, end of the cyclone.

As the fluid spirals inward from the periphery and approaches the radius of the vortex finder, its tangential velocity increases in accord with the principles of vortex action. The increasing tangential velocity, combined with the decreasing radius of revolution, results in a rapid increase in the centrifugal force tending to make the solids settle toward the wall. At the same time the radial components of flow tending to drag particles into the overflow are also increasing, owing to the convergence of flow as radial distance from the axis is diminished.

In a conventional liquid-solid cyclone the balance of these centrifugal and drag forces on particles is such that the resultant tendency to settle toward the wall is greater at smaller distances from the axis.

It is now assumed that the particle classification in a cyclone takes place at a size $D_{ps}$ which has at the radius of the vortex finder a settling velocity $u_{ps}$ equal in magnitude to the radial flow component $u_r$, since particles settling more rapidly at this radius could not be dragged within it and thence into the overflow. It is also assumed that the radial flow velocity component is constant at all points having a radial distance from the axis equal to the radius of the vortex finder. Thus the developments to follow will treat particles of the diameter of separation and flow conditions at the surface of an imaginary cylinder which is within the cyclone, concentric with the axis, and of the same diameter as the vortex finder.

Scale-up will be treated as the sum of two operations each of which can be represented by a change vector in the particle $C_R$ vs. $Re$ chart. These are, first, a scale-up of the system at constant cyclone Reynolds number in which the pressure drop across the cyclone will be allowed to change to maintain the constant $Re_{cyc}$, and, second, a change of cyclone pressure drop back to the original value.

As a first step, it will be shown that a change in pressure drop across a cyclone can be represented by an appropriate change vector in the particle $C_R$ vs. $Re$ chart. If pressure drop $p$ is the only cyclone variable changed independently, $\rho_p$, $\rho_f$, and $\mu$ will not be affected by the change. If $D_{ps}$ is designated the compensating variable, it is not varied in determining the $p$-change vector. Thus the effect of changing $p$ is to be evaluated in terms of changes in the remaining variables, $a$ and $u_{ps}$. Under the specified conditions for a cyclone pressure change vector,

since

$$C_R = \frac{4a\,(\rho_p - \rho_f)\,D_{ps}}{3u_{ps}^2\,\rho_f}$$

by definition

$$C_R \propto a/u_{ps}^2 \qquad (3)$$

since $\rho_p$, $\rho_f$, and $D_{ps}$ are not varied and since

$$Re = \frac{u_{ps}\,D_{ps}\,\rho_f}{\mu}$$

by definition

$$Re \propto u_{ps} \qquad (4)$$

since $\rho_f$, $\mu$, and $D_{ps}$ are not varied.

But

$$a \propto u_c^2 \qquad (5)$$

centrifugal force at radius of vortex finder

And at any given point in a cyclone, assuming uniform distributions of radial flow components,

$$u_r \propto q \qquad (6)$$

According to the measurements of Kelsall (4) in a given cyclone,

$$u_c \propto q^{1.115} \qquad (7)$$

empirical relationship

$$p \propto q^{2.14} \qquad (8)$$

empirical relationship

$$a \propto u_c^2 \propto q^{2.23} \propto u_r^{2.23} \qquad (9)$$

from (5), (6), and (8)

In accordance with the assumption, $u_{ps}$ is to be equal in magnitude to $u_r$ at the radius of the vortex finder. At this radius

$$C_R \propto u_r^{22.3}/u_{ps}^2 = u_{ps}^{2.23}/u_{ps}^2 = u_{ps}^{0.23} \qquad (10)$$

from (3) and (9)

$$C_R \propto Re^{0.23} \qquad (11)$$

from (10) and (4)

From (11) it follows that log $C_R$ varies as 0.23 log $Re$ when changes in cyclone pressure drop cause variations in $a$ and $u_{ps}$. This defines a direction on the particles $C_R$ vs. $Re$ chart.

A scale corresponding to the effect of changing $p$ may be derived as follows:

$$Re \propto u_{ps} \propto q \propto p^{1/2.14} \propto p^{0.47} \qquad (12)$$

from (4), (6), and (8)

or log $Re$ varies as 0.47 log $p$ along a pressure-change vector. Thus an axis can be drawn on a particle $C_R$ vs. $Re$ chart showing the direction and scale of a $p$-change vector. Provided that the model operation can be plotted on the $C_R$ vs. $Re$ curve, the effect of a change in cyclone operating pressure can be represented by the corresponding $p$-change vector, and its effect on separation size evaluated from a compensating $D_{ps}$-change vector.

The next step will be to show that the effect of scaling up the size of a cyclone at constant cyclone Reynolds number can also be represented by a corresponding change vector on the particle $C_R$ vs. $Re$ chart.

To the extent that gravitational forces may be disregarded in cyclone operation, the flow patterns in geometrically similar cyclone systems will be kinematically similar, with corresponding motions for corresponding particles, if operated at the same cyclone Reynolds number. In such cyclone systems, separation particle size would scale up in the same proportions as cyclone size. Therefore the change vector for cyclone size scale-up at constant cyclone $Re_{cyc}$ would be equal and opposite in direction to that for $D_{ps}$-change.

To maintain constant cyclone Reynolds number with constant fluid properties, velocity must decrease by the same change factor as the linear scale increases, as $Re_{cyc} = D_c u \rho_f/\mu$.

where

$D_c$ = linear scale of cyclone
$u$ = velocity at any given location
$Re_{cyc}$ = cyclone Reynolds number

From an energy balance it follows that the operating-head pressure must decrease by the square of the velocity change factor, since with dynamic symmetry, which also obtains at constant Reynolds number, the efficiency of conversion of pressure head to velocity remains constant. Thus when the cyclone is scaled up in size by a factor of $z$ at constant Reynolds number, the pressure $p$ drops by a factor of $z^2$

As a final step, scale-up of a cyclone at constant pressure may be evaluated as the sum of a scale up at constant Reynolds number, followed by a change in pressure back to its original value. Combining these change vectors analytically gives on a $D_c$-change vector

$C_R$ varies as $z^{-0.78}$
$Re$ varies as $z^{-0.06}$
$Re$ varies as $C^{0.08}$

These relationships give the scale and direction of a $D_c$-change vector. Provided that the model operation can be located on the particle $C_R$ vs. $Re$ curve, the effect of a change on cyclone scale can, within the limits of the approximations tacit in the preceding development, be represented by a corresponding $D_c$-change vector, and its effect on separation size evaluated from a compensating $D_{ps}$-change vector.

In order to plot the model operation on an appropriate particle $C_R$ vs. $Re$ curve, it has been assumed, rather arbitrarily, that the radial components are distributed uniformly along the cyclone length. The average radial flow velocity at the diameter of the vortex finder was then calculated from cyclone geometry and feed flow. The separation size $D_{ps}$ is known from the model operation, while the density and viscosity of the suspending liquid, water, were used as those of the medium. It will be recognized that this entails many approximations, but it will also be apparent from the shape of the $C_R$ vs. $Re$ curve that a very considerable error in evaluating $Re$ will not change the shape of this vector construction much and hence should not lead to a serious error in scale-up.

Although methods based on the one given have been used successfully in predicting scale-up of liquid cyclones, it should be noted that considerations beyond the scope of this paper enter in most cases. For example, the $C_R$ vs. $Re$ curve shown in Figure 1 is valid only for spherical particles at infinite dilution, where particle diameter is taken as the measure of particle "size" or linear scale. At finite dilutions the curve would follow a different course, because of particle interferences. The curve for irregular particles would follow yet a different course, because of the influence of particle shape on settling rate, and the exact position of the plotted curve will further depend upon just what linear dimension of the particles is taken as a measure of its scale or size. (Dimensional analysis permits use of any characteristic linear dimension.) Thus, as textbooks show, there will be a family of $C_R$ vs. $Re$ plots for varying particle shapes and dilutions. Cyclone scale-up constructions must be built on an appropriate one of the family. Also the particle system is usually not scaled up in toto as required for geometric symmetry of the systems, and relative particle size has an effect on separation (*3*).

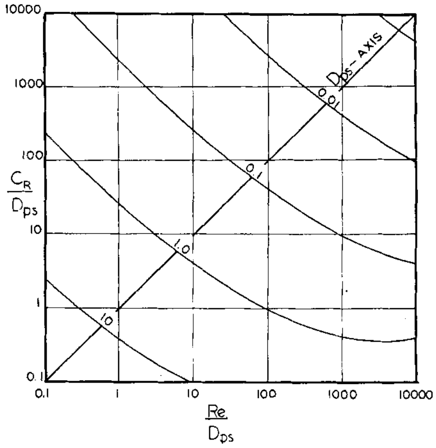


Fig. 5. Parametric representation of $D_{ps}$.

## PARAMETRIC REPRESENTATION

It often happens, as in cyclone scale-up problems, that some one of the variables will usually appear as the dependent or unknown one. In such cases a parametric representation of this variable is convenient.

If one subordinate variable, such as $D_{ps}$, has a fixed value, it follows that all summation points incomplete by the vector for this variable will be equidistant from the appropriate $C_R$ vs. $Re$ curve in a direction corresponding to that of the chart vector for the missing variable. The locus of all incomplete summation points for a given value of $D_{ps}$ is therefore a curve of the same shape as the $C_R$ vs. $Re$ curve, but displaced along the $D_{ps}$ axis by a distance corresponding to the fixed value of $D_{ps}$.

The summation of all variables except $D_{ps}$ gives a component in the $C_R$ direction (on a log scale) which is equal to that of $C_R/D_{ps}$ and a component in the $Re$ direction equal to that of $Re/D_{ps}$. Therefore, the locus of all points having a given value of $D_{ps}$ can be plotted on a graph of log $C_R/D_{ps}$ vs. log $Re/D_{ps}$ (Figure 5) for various values of $D_{ps}$ as parameter. The chart of Figure 5 is similar in general principle to the Fontein velocity chart described by Driessen (*2*) and might be considered a transformation from Fontein's chart. Its merit is that it permits subordinate axes to be assigned to all independent subordinate variables, while Fontein's representation of the same relationship does not. Constructions as previously described can be made on Figure 5 if $D_{ps}$ is the dependent variable, but the length of the compensating $D_{ps}$ vector is here indicated directly by the parametric $D_{ps}$ scale. Obviously analogous parametric charts could be made with any other variable as the dependent one.

## NOTATION

(Self-consistent units to be used throughout)

$a$ = accelerating force acting on settling particle
$C_R$ = drag coefficient for settling particle, a dimensionless number equal to $4a(\rho_p - \rho_f)D_p/(3u_p^2\rho_f)$
$D_c$ = linear scale (diameter) of cyclone
$D_p$ = linear scale (diameter) of particle
$D_{ps}$ = linear scale (diameter) of particle at which size separation is made
$n$ = integral exponent
$p$ = operating pressure drop across cyclone
$q$ = feed rate of slurry to cyclone
$Re$ = Reynolds number for settling particles, a dimensionless number equal to $u_p D_p \rho_f/\mu$
$Re_{cyc}$ = Reynolds number for cyclone, a dimensionless group equal to $D_c u \rho_f/\mu$
$u$ = any characteristic velocity in a cyclone
$u_c$ = tangential component of flow velocity at any point in a cyclone
$u_p$ = terminal settling velocity of particle
$u_{ps}$ = terminal settling velocity of particle at which size separation is made
$u_r$ = radial component of flow velocity in cyclone at radius of vortex finder
$z$ = scale-up factor
$\rho_f$ = fluid density
$\rho_p$ = particle density
$\mu$ = fluid viscosity
$\propto$ = varies as

Superscript (′) denotes fixed value for variable.

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